CALCUTTA MATHEMATICAL SOCIETY

AE 374, Salt Lake (Sector I), Kolkata 700 064

Mathematical Ability Test : Sample Question

Try to answer as many questions as you can

Section A (MCQ Type) / Full Marks 40

All questions carry equal marks

Put a tick (√) mark against your answer

(No negative marking for wrong answer)

All questions carry equal marks

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1. **The number of real roots of the equation**
   $|1-x| + x^2 = 5$ is
   (A) 1 (B) 2 (C) 4 (D) 0
2. **A square is cut into two rectangles such that the sum of the perimeters of the two rectangles is 48 cm; then the side of the square is (in cm)**
   (A) 5 (B) 8 (C) 6 (D) None of the above
3. **Given (x+2) and (2x-1) are factors of ($2x^3 + ax^2 + bx + 10$) then (a^2+b^2) is equal to**
   (A) 338 (B) 218 (C) 74 (D) none of the above.
4. **The sum of ten distinct real numbers is 50. Then the sum of their squares is :**
   (A) can be 50 (B) is 100 (C) less than 250 (D) is always greater than 250
5. **Let x, y be positive integers such that $(x + y)^2 - 2 (xy)^2 = 1$**
   If z=x+y, then z equals to
   (A) 2 (B) 3 (C) 4 (D) 5

6 . A person is paid Rs 150 for each day he works and is fined Rs 30 for each day he is absent. If in 40 days his net earning is Rs 3300 /- then the number of days he was absent is

(A) 20 (B) 10 (C) 15 (D) 12

7. A two wheeler company increased its production of a particular brand from 80000 to 92610 in three years. The rate of growth for the particular brand is

(A) 6% (B) 4% (C) 4.5 % (D) 5%

8. A thinks of a positive integer which B doubles and C trebles the B's number. Finally D multiplies C's number by 6. E notices that the sum of the four numbers is a perfect square. The smallest number that A could have thought of is

(A) 3 (B) 2 (C) 4 (D) 5

9. Ram and Rahim individually can complete a work in 15 days and 20 days respectively. They jointly completed another work in 30 days. Their earning ratio from the work should be

(A) 5:4 (B) 4: 3 (C) 2: 3 (D) none of the above

10. p and q are two distinct prime numbers (p>q) greater than 5. then $p^2 - q^2$ is

(A) always divisible by 6 but not divisible by 12

(B) always divisible by 12 but not divisible by 24

(C) always divisible by 24 but not necessarily divisible by 48

(D) none of the above.

Section B (Short Answer Type)
Full Marks 80

All questions carry equal marks
Give all relevant steps for all answers.

(1) How many positive integers less than 2015 may be written as a sum of two consecutive positive integers and also can be written as sum of five consecutive positive integers.

(2) A positive integer p can be written as $m^2 + 3n^2$, where m and n are positive integers. Another positive integer q can be written as $q = c^2 + 3d^2$, where c and d are positive integers. Show that pq also can be written in the same form i.e. pq can be written as $pq = r^2 + 3s^2$ where r and s are positive integers.

(3) Which is greater $(31)^{11}$ or $(17)^{14}$? Give reasons for your answer (Hint: replace 31 by 34)

(4) Solve for x, y, z

$$x + y + z = 14$$
$$x^2 + y^2 + z^2 = 84$$
$$xy = z^2$$

(5) Find the smallest number which when divided by 3, 5, 7, 11 leaves remainder 2, 4, 6, 1 respectively.

(6) A cone with circular base is cut into two sections by a horizontal plane parallel to the base in such a manner that the curved surfaces of the two sections are of equal area. If the height of the original cone is 15 cm find the height of the smaller cone remained after the cut.

(7) Find the value of the sum

$\cos(\pi/1000) + \cos(2\pi/1000) + \ldots\ldots\ldots + \cos(999\,\pi/1000)$

(8) Two circles of radii r, s touch each other at a point. A common tangent (other than the one at A) touches the circles at P and Q respectively. Show that $PQ^2 = 4rs$

(9) If f(x) is a polynomial with integer coefficients and f(1) and f(2) are both odd then prove that there exists no integer n for which f(n) =0

(10) Suppose m, n are integers both greater than 3 and $m = n^2 - n$. Show that (m^2-2m) is divisible by 24.

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